university of groningen

Exam Computational Fluid Dynamics (CFD)

January 21st 2016 9.00-12.00

This exam is 'closed book'. It is NOT allowed to use a book or lecture notes. Only the use of a simple pocket calculator is allowed. Programmable calculators are not allowed, nor the use of electronic devices (tablet, laptop, etc.).

Always give a clear explanation of your answer. An answer without any computation will not be rewarded, so also copy the computations from your scratch paper.

Write your name and student number on each page!

Free points: 10

Practica: 30 For each of the 5 practica a maximum of 6 points can be earned.

1. The unsteady convection diffusion equation for $0 \leq x \leq 1$ is given by

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = k\frac{\partial^2 \phi}{\partial x^2},$$

with mixed Dirichlet-Neumann boundary conditions $\phi(0,t) = 0$, $\partial\phi/\partial x(1,t) = 0$ and initial condition $\phi(x,0) = x$. The flow is characterised by $U = 4$ and $k = 0.05$.

   (a) 12
   (1) Use 2nd order central discretisation (for both convection and diffusion) and 1st order explicit time discretisation to compute the solution on an equidistantly spaced grid with only one internal point after two time steps of $\delta t = 0.1$.
   (2) Does the solution contain wiggles? Explain.
   (3) Is the computation stable (for $t \to \infty$)? What is the maximum allowed time step?
   (4) When 1st order upwind is used for convection, does the quality of the solution change regarding accuracy, wiggles, stability? Explain.

   (b) 6
   (1) Use the same grid and time steps as in (a) and use again central spatial discretisation, but now in combination with 1st order implicit time discretisation to compute the solution after two time steps.
   (2) Does the solution contain wiggles? Explain.

2. 8 Consider 2nd order upwind (B3) discretisation for the first derivative

$$\frac{\partial \phi}{\partial x} = \frac{1}{\delta x}\left(\frac{3}{2}\,\phi_i - 2\,\phi_{i-1} + \frac{1}{2}\,\phi_{i-2}\right)$$

which is obtained from the λ-schemes for $\lambda = 1/2$.
Derive for which Peclet numbers the solution of the steady convection-diffusion equation will be wiggle-free, when B3 is used for convection and standard 2nd order central discretisation for diffusion. Hint: try fundamental solutions of the form r^i and monitor the sign of r, or (alternatively) use the concept of positive coefficients.

P.T.O.

3. (a) [12] Consider the steady convection-diffusion equation on $0 \leq x \leq 1$ with velocity U=1 and Dirichlet boundary conditions $\phi(0) = 0$ and $\phi(1) = 1$.
   (1) Use both the discretisation methods A and B and solve the discrete systems on a non-uniform grid with only one interior point at $x = 1 - 2k$ (k remains variable).
   (2) Sketch both solutions using linear interpolation between the grid points.
   (3) Compute the exact solution (at $x = 1 - 2k$) as well. Which discrete solution is better? Does this depend on the magnitude of k?

   (b) [6] Show and explain the favourable properties of method A regarding symmetry and eigenvalues.

4. (a) [6] Describe the actions to be performed at one time step during an explicit time-stepping method to solve the incompressible unsteady Navier-Stokes equations. Formulate the required (discretised) equations (in vector form for 'easy writing').

   (b) [4] Explain why $\nabla \cdot u^n$ should be retained in the discretised equations, although it is close to zero. Derive the formula that describes the accumulation of errors.

5. (a) [3] Suppose that for a ship the Reynolds number corresponding to the largest scales is approximately $Re_{ed} \approx 10^8$. For a 3D DNS at least ${Re_{ed}}^{9/4}$ grid-points are required, which follows from Kolmogorov theory. Suppose further that, each time step, the Navier-Stokes solver requires 10^5 floating point operations (flops) per grid-point. How long does a DNS of 1 second real-time take, with time steps of $\delta t = 10^{-4}$, on today's fastest parallel computer, which can handle (almost) 10^{18} flops per second?

   (b) [3] Give 3 items that will speed up a DNS as described in a).

Total: [100]